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# The Grading Entropy-Based Criteria for Structural Stability of Granular Materials and Filters

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Additional information is available at the end of the chapter

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## Abstract

Some rules for particle migration, filtering, and segregation were elaborated on the basis of some simple laboratory tests and data of well-designed, artificial mixtures of natural sand grains. Use was made of the knowledge available in the field and two pairs of grading entropy parameters. These parameters incorporate all information of the grading curve and are pseudo-metrics in the “space of the possible grading curves.”

**Keywords:** grading entropy, internal erosion, suffosion, filtering, segregation, piping

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## 1. Introduction

The internal stability of compacted earth dam materials, granular filters, and soils on natural slopes is essential. The internal erosion involves loss of particles under seepage flow; the matrix of coarse soil particles may or may not be unstable [1–3]. The term “suffosion” is Russian in origin and is used to describe the process of removal and transport of small soil particles through pores [4, 5].

It is desirable that adjacent materials in earth dams or rockfill dams should act as filters for each other and the material should not segregate [6–14]. Broadly graded materials may segregate during the construction process where the particles are able to flow freely, such as tipping and spilling. The likelihood of backward erosion is greater for segregated soil than for non-segregated soil [7, 8].

The inherent stability or proneness to segregation is usually specified in terms of particular diameters  $D_x$  (or  $d_x$ ), which represent the particle diameter for which  $x\%$  of grains (by weight)

are smaller. The susceptibility to suffosion is assessed by the graphical approach [5], where a grain size distribution is compared with empirical upper- and lower-bound thresholds; the method is not valid for gap-graded grading. The filter rules—the compacted earth dam or core material should obey when associated with each other and with the dam base—are formulated in terms of pairs of grading curve points [9–15], which is “too simple” in case of broadly graded soils.

This chapter summarizes three grading entropy-based rules, on the basis of the original work of Lőrincz and some applications [15–22]. The suggested rules differ from most existing rules in that the whole grading curve is used instead of some limited number of grading curve points, without any constraint on the shape of the grading curve. They were elaborated on the basis of the knowledge available, the measured data available in the literature, and data measured for well-designed sand mixtures by Lőrincz. The rules were verified by the examined cases [19–22], an example included.

## 2. Grading curve characterization

The grading curve is a statistical distribution of logarithm of the diameter with respect to the dry weight. It is a discrete distribution curve with a non-uniform cell system in arithmetic scale. To characterize it, first of all, the statistical cell system—the so-called abstract fraction system—is defined and the space of the grading curve is introduced.

Then the two grading entropy parameter pairs are introduced. The first pair is related to the expected (log diameter) value of the grain size distribution, in non-normalized and normalized forms. The normalized version has a shift symmetry on the log  $d$  axis.

The second pair is the entropy arisen from the mixing of the fractions, in non-normalized and normalized forms. Its maximum for a fixed value of the first coordinate is related to a single grading curve with finite fractal distribution. The grading curves can be represented in terms of the two parameters in the entropy diagram.

### 2.1. The fractions

The fraction system is defined on the pattern of the classical sieve hole diameters (where measurements are made), by successive multiplication with a factor of 2, as follows. The diameter range for fraction  $j$  ( $j = 1, 2, \dots$ , see **Table 1**):

$$2^j d_0 \geq d > 2^{j-1} d_0, \quad (1)$$

or the upper diameter range for fraction  $j$ :

$$d_j = 2d_{j-1} \quad (2)$$

Using  $\log_2$  form results in an integer increment by each multiplication and fraction as follows:

$$\log_2 d_j = 1 + \log_2 d_{j-1} \quad (3)$$

Fraction number $j$	1	...	23	24	...
$d$ [-]	$d_0$ to $2 d_0$	...	$2^{22} d_0$ to $2^{23} d_0$	$2^{23} d_0$ to $2^{24} d_0$	...
$d$ [mm]	$2^{-22}$ to $2^{-21}$	...	1–2	2–4	...
$S_{0j}$ [-]	1	...	23	24	...

**Table 1.** Definitions of fractions, based on the smallest particles likely to occur in nature.

The variable  $d_0$  is the arbitrary smallest diameter, and assumingly a 2-power,  $d_0 = 2^{-k}$ . Its possible value is equal to the height of the  $\text{SiO}_4$  tetrahedron ( $d_0 = 2^{-22}$  mm).

$$\log_2 d_1 = 1 + -22 \quad (4)$$

The fraction serial number variable can be expressed by the diameter:

$$j = k + \log_2 d_j \quad (5)$$

The integer  $j/j - 1$  is a so-called abstract upper/lower diameter limit ( $\log_2 d_j$  shifted by  $k$ ).

## 2.2. The grading curve space

By the measurements of the fractions during sieving, the relative frequencies of the fractions  $x_j$  ( $j = 1, 2, 3 \dots N$ ) can be determined. These fulfill the following equation of each grading curve:

$$\sum_{j=1}^{\infty} x_j = 1, x_j \geq 0 \quad (6)$$

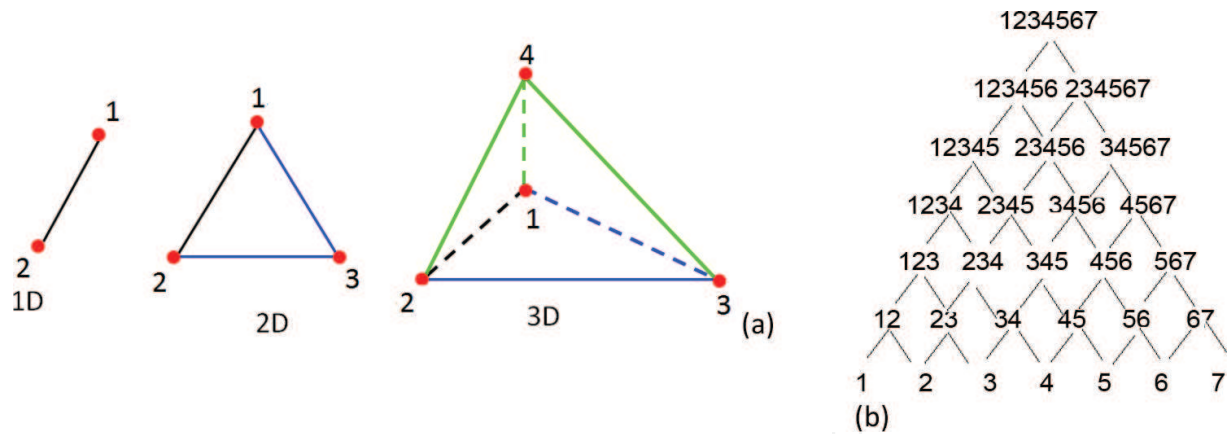
which can be rewritten as follows:

$$\sum_{i=1}^N x_i = 1, x_i \geq 0, N \geq 1. \quad (7)$$

where  $i$  is a rescaled fraction serial number being equal to 1 at the finest non-zero fraction with original serial number  $j_{\min}$  (see **Table 1**), the integer variable  $N$  is the number of the fractions between the finest  $j_{\min}$  and coarsest  $j_{\max}$  non-zero fractions:

$$N = j_{\max} - j_{\min} + 1. \quad (8)$$

The space of the grading curves with  $N$  fractions can be identified with the  $N - 1$ -dimensional, closed simplex (which is the  $N - 1$ -dimensional analogy of the triangle or tetrahedron, the two- and three-dimensional instances, **Figure 1**) as follows. Each grading curve is related to a simplex point, the relative frequencies  $x_i$  can be identified with the barycentric coordinates of the points of the  $N - 1$ -dimensional closed simplex.



**Figure 1.** (a) Standard simplex images with dimension less than 4. (b) The lattice of the continuous sub-simplices of the six-dimensional simplex.

### 2.3. The grading entropy coordinates of the grading curve

The grading entropy concept is an application of the statistical entropy to the grading curve [15, 23], by introducing a uniform cell system for the derivation besides the fractions. It condenses the information of the whole grading curve into two pairs of parameters. The grading entropy  $S$  is the sum of two “means” [15]:

$$S = S_0 + \Delta S \quad (9)$$

which are called as base entropy  $S_0$  and entropy increment  $\Delta S$ . The base entropy  $S_0$  is a weighted mean or expected value of the fraction serial number:

$$S_0 = \sum_{i=i_{\min}}^{i_{\max}} x_i S_{0i} \quad (10)$$

which depends linearly on the  $\log_2$  diameter  $d$  (see Eq. (5)),  $S_{0i}$  is the grading entropy of the  $i$ th fraction, being identical to the fraction serial number (see **Table 1**):

$$S_{0i} = i \quad (11)$$

Any decrease in the base entropy  $S_0$  can be explained by the decrease of the mean grain diameter, for example, due to breakage. Any increase in the base entropy  $S_0$  can be explained by the increase of the mean grain diameter, for example, due to suffosion or segregation.

The relative base entropy  $A$  is defined as follows:

$$A = \frac{S_0 - S_{0\min}}{S_{0\max} - S_{0\min}} \quad (12)$$

where  $S_{0\max}$  and  $S_{0\min}$  are the entropies of the largest and the smallest fractions in the mixture, respectively.

The relative base entropy  $A$  varies between 0 and 1, its value is equal to 0.5 if all relative frequencies of the fractions  $x_j$  ( $j = 1, 2, 3, \dots, N$ ) are equal. It measures the distance of the mean  $\log_2$  diameter and the smallest  $\log_2$  diameter. Geometrically, the grading curves with the same  $A$  have the same sub-graph area. Since  $A$  is linear, the  $A = \text{constant}$  condition in addition means parallel hyperplanes in the  $N - 1$ -dimensional space generated by the simplex (**Figure 2(a) and (b)**).

The entropy increment  $\Delta S$  is the logarithm of the weighted generalized geometric mean of the relative frequencies of the fractions  $x_j$  ( $j = 1, 2, 3, \dots, N$ ):

$$\Delta S = -\frac{1}{\ln 2} \sum_{x_i \neq 0} x_i \ln x_i. \quad (13)$$

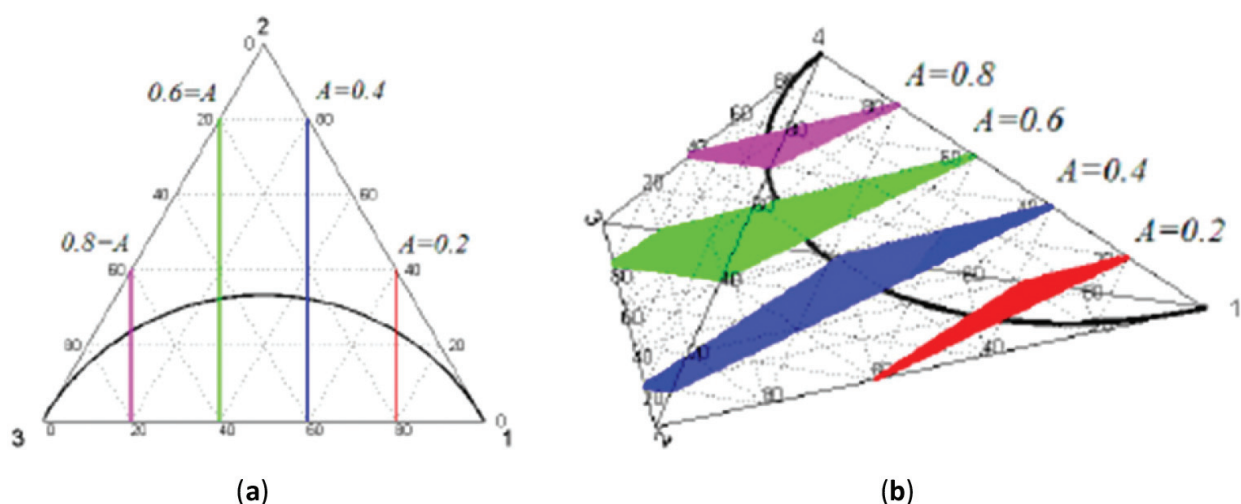
The entropy increment  $\Delta S$  measures how much the soil behavior is really influenced by all of its  $N$  fractions. For those grading curves, in which all  $N$  fractions are well represented, the entropy increment is typically close to  $\ln N / \ln 2$ . The normalized entropy increment  $B$  is defined as follows:

$$B = \frac{\Delta S}{\ln N} \quad (14)$$

Being a strictly concave function, the normalized entropy increment  $B$  has a unique maximum for each  $A = \text{constant}$  value, which is the following grading curve with finite fractal distribution (see the definition in [24]):

$$x_1 = \frac{1}{\sum_{j=1}^N a^{j-1}} = \frac{1-a}{1-a^N}, \quad (15)$$

$$x_j = x_1 a^{j-1} \quad (16)$$



**Figure 2.** The constant  $A$  sections of the simplex and the optimal line (a) for a two-dimensional simplex and (b) for a three-dimensional simplex [16].

where parameter  $a$  is the root of the following equation:

$$y = \sum_{j=1}^N a^{j-1} [j - 1 - A(N - 1)] = 0. \quad (17)$$

As  $A$  varies between 0 and 1 (the extreme values represent the extreme fractions 1 and  $N$ ), the positive root  $a$  varies between 0 and  $\infty$  in the function of  $N$ . The optimal grading curve has finite fractal distribution with fractal dimension  $n$  given by:

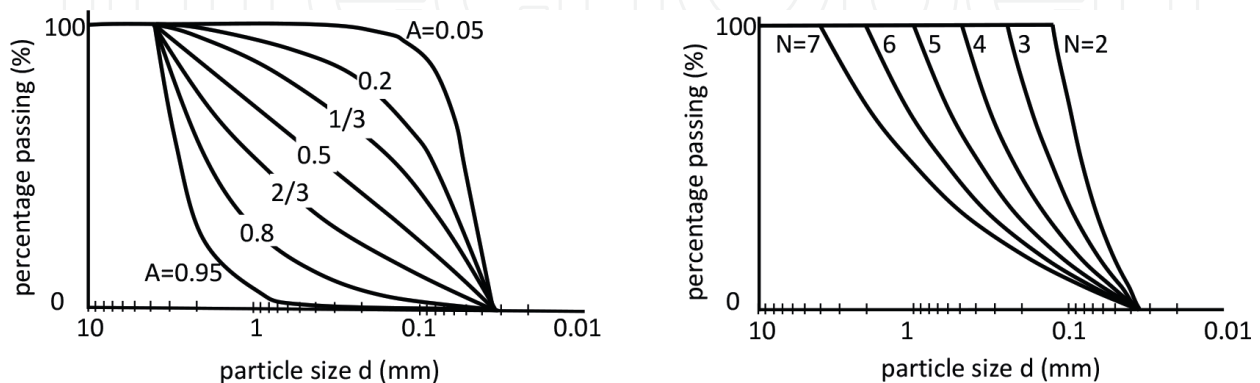
$$a = 2^{(3-n)} \quad (18)$$

The optimal grading curve is concave if  $A < 0.5$ , linear if  $A = 0.5$ , convex if  $A > 0.5$  (see **Figure 3**). Having no inflexion points, the optimal grading curve has the shortest curve length out of the possible grading curves with a specified  $A$ . The optimal points of the simplex constitute a continuous line called optimal line which can be seen in **Figure 2**.

In the linear case, it has a unique maximum, being equal to  $1/\ln 2$ , in the center of the simplex where each relative frequencies  $x_i$  are the same, in this case the “disorder” is maximal in the system. The disorder originated from mixing of the fractions can be measured by the entropy increment  $\Delta S$ , which is the entropy of the fractions neglecting the fact that the width of the statistical cells in arithmetic scale is different

## 2.4. The entropy diagram

Four kinds of maps can be defined between the  $N - 1$ -dimensional simplex with fixed  $N$ , with fixed smallest fraction serial number  $i_{\min}$  and the two-dimensional space of the entropy coordinates: entropy map with coordinates  $[S_o, \Delta S]$ , the normalized entropy map with coordinates  $[A, B]$  and the two partly normalized entropy maps with a mixture of normalized and non-normalized coordinates, that is,  $[S_o, B]$  or  $[A, \Delta S]$ . The optimal line of the simplex—between vertices 1 and  $N$ —



**Figure 3.** Optimal grading curves with finite fractal distribution, (a)  $N = 7$ ,  $A$  varies. (b)  $A = 2/3$ ,  $N$  varies.

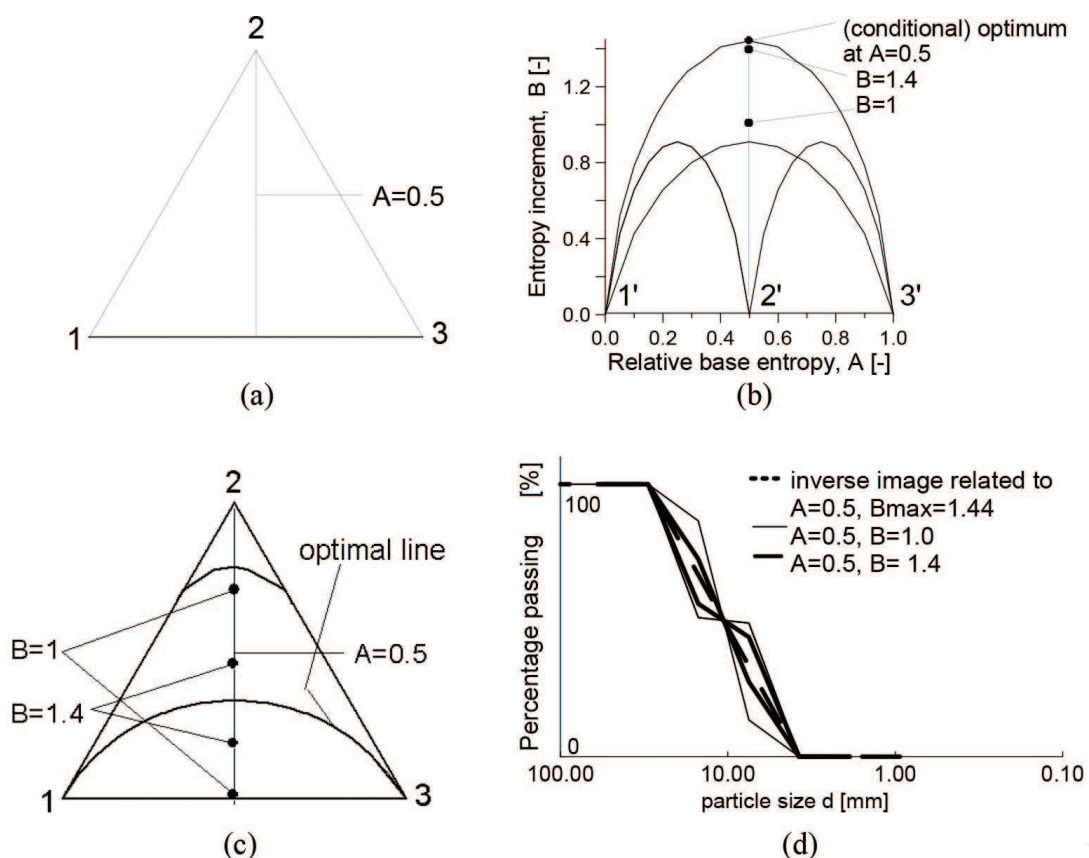


These maps are continuous on the closed simplex for fixed  $N$ . The image of the simplex—the entropy diagram—has a maximum and a minimum value for every possible value of  $A$  or  $S_o$ . The optimal line of the simplex maps into the maximum line, the map along the optimal line—maximum  $B$  line is one to one (see **Figure 4(c)** and **(d)**). The simplex edges and vertices map into the minimum line. The minimum and maximum diagram lines—illustrated in **Figures 4** and **5**—differ in scaling.

In terms of the original entropy coordinates, the map is continuous if  $N$  is changing, and in terms of the normalized entropy coordinates, the map is not continuous if  $N$  is changing. The normalization with respect to a coordinate results in the range being fixed in that direction. The maximum  $B$  point is equal to  $1/\ln 2 = 1.44$  for any  $N$ , the maximum of the entropy increment  $\Delta S$  is equal to  $\ln N/\ln 2$ .

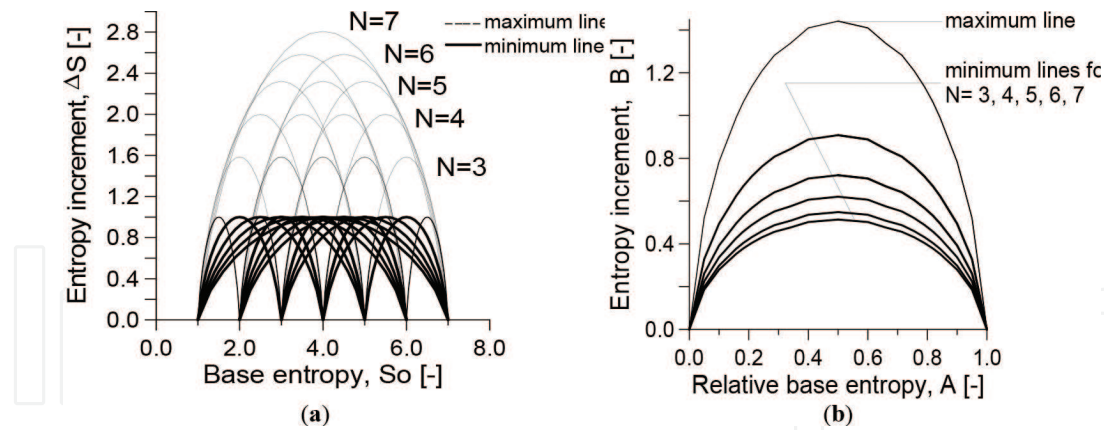
The images of the optimal lines—the maximum  $B$  lines—will nearly coincide for any number of the fractions  $N$  (**Figure 5(b)**) in the normalized diagram but will separate in terms of the non-normalized coordinates  $[S_o, \Delta S]$ , reflecting the structure of the continuous sub-simplexes of a simplex. This is illustrated in **Figures 1(b)** and **5(a)** for soils with up to seven fractions.

The inverse image of a regular normalized entropy diagram point  $[A, B]$  in the simplex is situated on the  $A = \text{constant}$ ,  $N - 2$ -dimensional, affine hyperplane, with shape of an  $N - 3$ -dimensional



**Figure 4.** The normalized entropy map and the inverse image for  $N = 3$ . (a) The two-dimensional simplex and its  $A = 0.5$  hyperplane section. (b) The image of the simplex in the entropy diagram. (c) and (d) The inverse image of points  $A = 0.5$ ,  $B = 1$ ,  $1.4$  and  $B_{max} = 1.44$  in the simplex and in the space of the grading curves, respectively.





**Figure 5.** Diagrams of simplexes with various  $N$ , the image of the optimal lines (“maximum lines”) and the image of the edges  $1 - N$ ,  $N = 2-7$  (i.e., “minimum lines”). (a) The non-normalized diagram. (b) The normalized diagram.

“sphere,” centered to the optimal point, its “radius” depends on  $B_{max} - B$ . The related grading curves have the same sub-graph area and the deviation from the optimal grading curve depends on  $B_{max} - B$ .

### 3. The construction of the grading entropy-based rules

#### 3.1. The methods

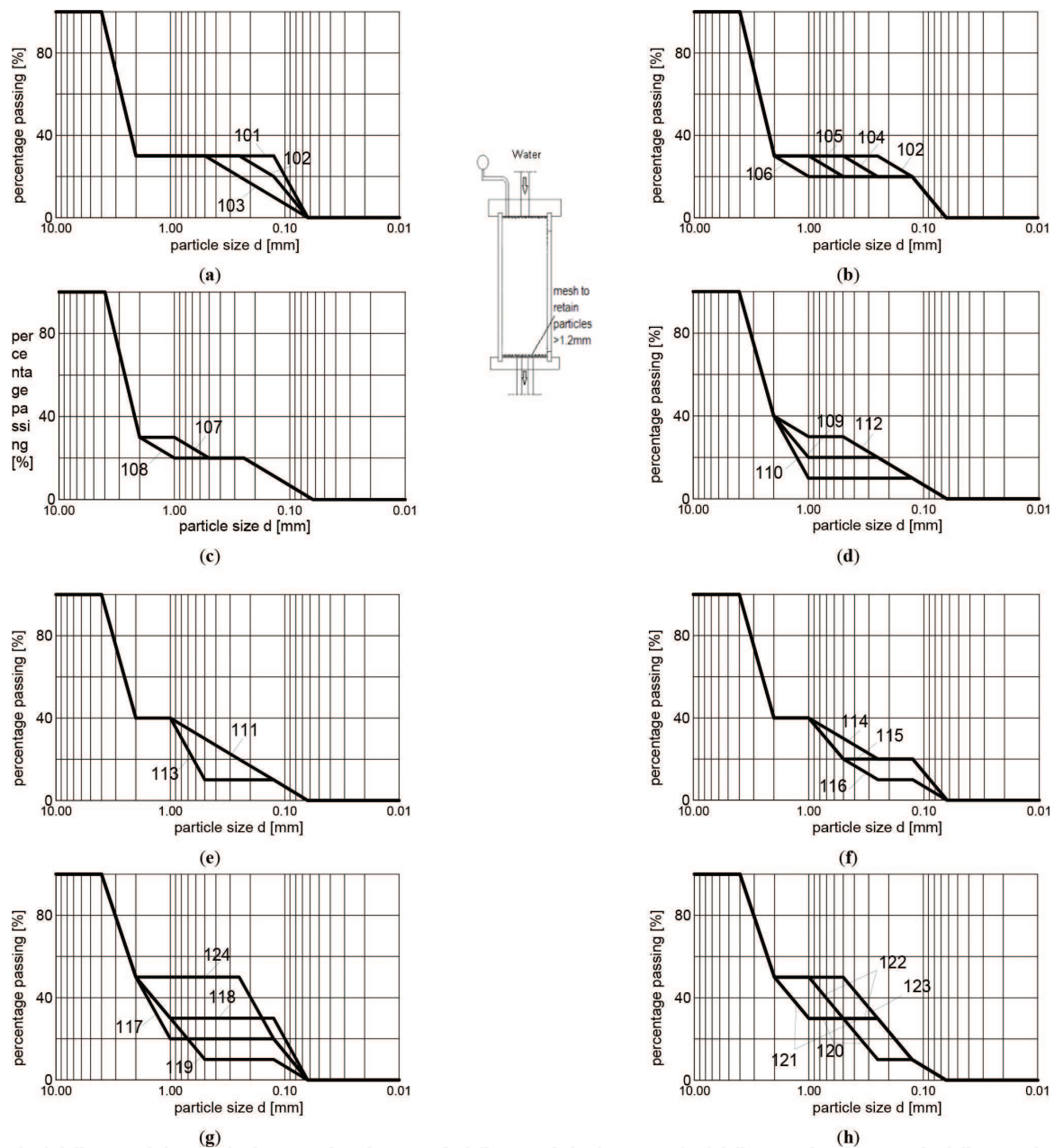
The particle migration (or internal stability) rule, the filter rule, and the segregation rule were constructed as follows. For each rule, simple soil testing programs were designed and executed by using artificial mixtures of natural sand grains [15]. Two variables were carefully constructed using the grading entropy concept [15] for each rule separately.

The entropy variables were used such that the experimental data were plotted on diagrams, differentiating points which exhibited different physical behavior so that domains of particular behaviors could be defined. In addition, some other information and existing data (e.g. the data base related to Ref. [7]) were used.

The additional information was as follows: one piece of information used was that there could be no more than two empty particle size fractions between the filter and the base soil, before the base soil cannot be retained by the filter [21]. This can be derived using Pure Geometry Theorems [21] and also by using the Terzaghi filter criterion (i.e., the finer is to be protected) as follows:

$$1 \leq \frac{D}{d} \leq 4 \quad (19)$$

where  $D$  and  $d$  are the diameter of the filter and the base soil. Another piece of information was that the data of suffosion tests could be used for the filter rules, and vice versa, based on the following consideration which comes from the self-filtering theory of Kézdi [25, 26]. If the gap-graded grading curve (see eg. **Figure 6**) is cut into two parts, with the coarser part



**Figure 6.** (a–h) Some grading curves of samples used by Lőrincz [15, 21] for suffosion tests. The inset shows the permeameter test arrangement used in the tests.

considered as the filter and the finer part as the base soil, and the base soil part being filtered, then there will be no suffosion within the soil.

### 3.2. Particle migration rule

For the particle migration (internal stability-suffosion) rule, simple vertical flow tests were designed and executed using artificial mixtures of natural sand grains. The dimensions of the permeameter were 20 cm in height and 10 cm in diameter. It was closed at the bottom by a sieve which was permeable of grains smaller than 1.2 mm but which retained grains larger than 1.2 mm.

The downward hydraulic gradient  $[i]$  was between four and five. The two parts of the permeameter were separated after the test, and the grading curves were determined. The grain movements were detected.

The results of the vertical water flow (suffosion) test were represented in the partly normalized entropy diagram, in terms of the relative base entropy and the entropy increment coordinates  $A$  and  $\Delta S$ , as shown in **Figure 7(a)** and the rule was separately set up for each specified  $N$  value. The part of the diagram where the gap-graded grading curves resulted in self-suffosion was indicated by letter  $b$ . The simplified normalized diagram version is shown in **Figure 7(b)**.

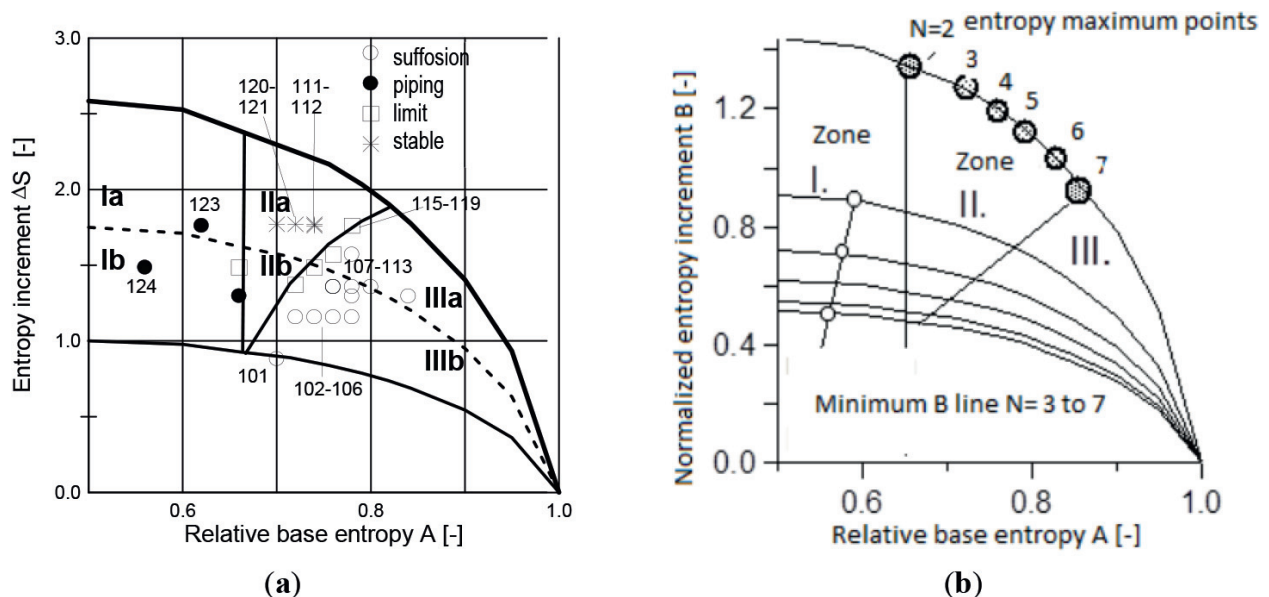
If  $A < 2/3$  (zone I), the soil was internally unstable; if  $A = 2/3$  and  $A < 2/3$  (zones II, III), the soil was internally stable. In zone II, there were no particle movements: the larger particles retained the smaller particles and the smaller particles supported the larger ones. In zone III, the fines may have migrated in the presence of seepage flow ("suffosion").

The internal stability zone III was separated by the  $2/3$  vertical line. The division curve between zones II and III connects the maximum entropy points of the mixtures with fraction number less than  $N$ , the shape of the curve between II and III is approximate in **Figure 7(b)**.

The rule can be interpreted such that, in zone I (where  $A < 2/3$ ), no structure of the larger grains is present, the coarse particles "float" in the matrix of the fines and become destabilized when the fines are removed by piping. In the zone where  $A = 2/3$  and  $A > 2/3$  the coarse particles form a skeleton and total erosion cannot occur. In zone III, the structure of larger particles is inherently stable, the smaller particles may move by suffosion.

### 3.3. Filter rule

The filter rule was developed using three series of tests: the filter tests of Sherard [7], the filter tests of Lőrincz [15], and the suffosion tests of Lőrincz [15]. The grading of the soils tested by



**Figure 7.** (a) Particle migration zones in half of the partly normalized entropy diagram for mixtures with  $N = 6$  fractions, the three digit numbers are related to the grading curves shown in **Figure 6**. (b) Particle migration zones in the simplified, normalized entropy diagram [21].

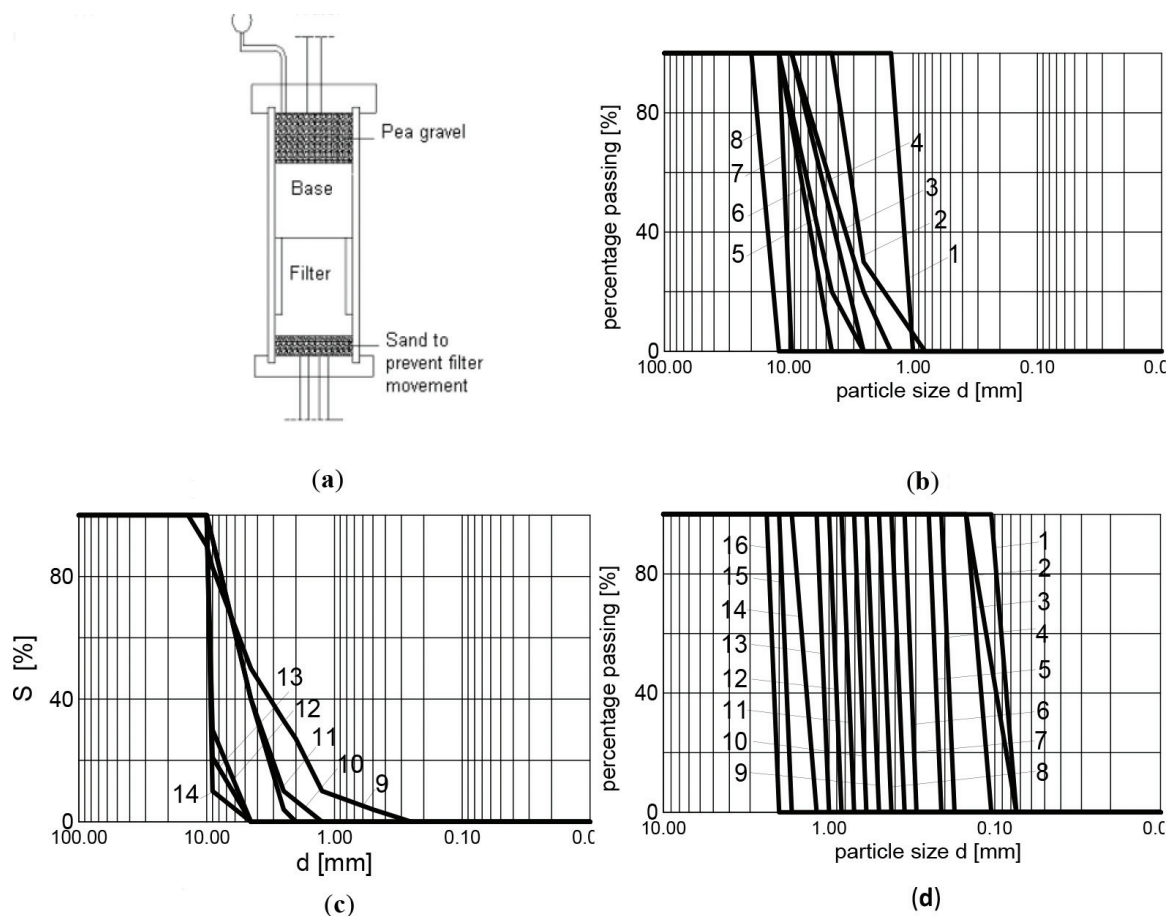
Sherard is shown in **Figure 8** and the grading of the soils tested by Lőrincz is shown in **Figures 6 and 9**. In the filtering test, the filter and base soils are placed into the permeameter (20 cm in height and 10 cm in diameter) in series separated by a sieve. The downward hydraulic gradient [*i*] is between four and five. The suffosion tests of Lőrincz [15] were reanalyzed as follows. The gap-graded grading curves shown in **Figure 6** were cut into two parts at the gap and one part was considered as the filter and the other part as the base soil. If suffosion occurred, then the filter was unsafe.

Two pseudo-metrics were constructed from the grading entropy parameters. The logarithm of the difference between base entropies of the filter and base soils,  $\log(S_{0f} - S_{0b})$  described the distance between the mean  $\log_2$  diameters of the filter and base soils. The sum of filter and base soil entropy increments  $\Delta S_f + \Delta S_b$  expressed the sum of the two “effective” fraction number or *N* values (i.e., the total number of important fractions in the two grading curves).

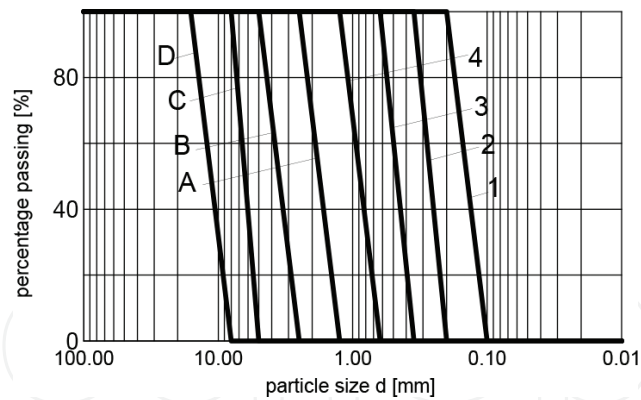
Plotting the test results in terms of the foregoing variables, the safe and unsafe areas were separated by a straight line: a layer acts as a filter for an adjacent layer (for the base soil) on the condition that:

$$\Delta S_f + \Delta S_b \geq 4 \ln(S_{0f} - S_{0b}) - 4.39 \quad (20)$$

where index *f* and *b* denote the filtering layer and the material being filtered (base soil), respectively. The domains defined by the Eq. (20) are shown in **Figure 10**. The point where



**Figure 8.** (a) Filter test apparatus in [7, 21]; (b) and (c) Sherard-filter soils [7, 21]; (d) Sherard-base soils [7, 21].

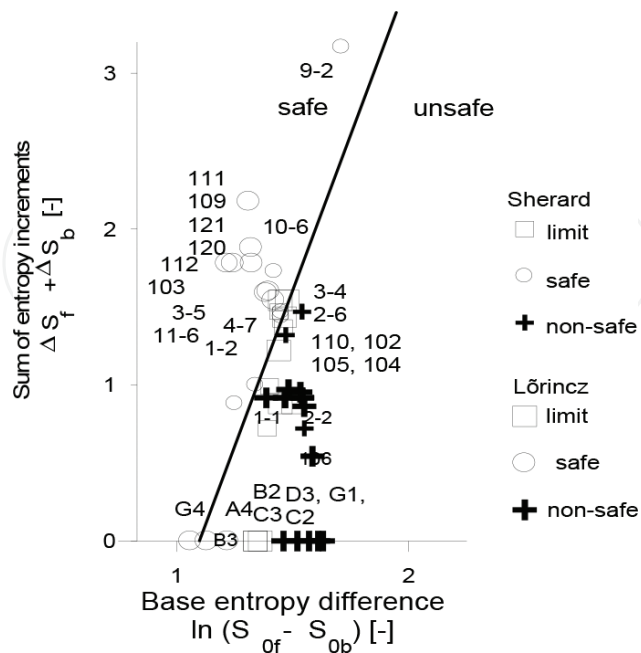


**Figure 9.** Some grading curves of soils used in the filter tests of Lőrincz [15, 21]. Note that the filters are identified by characters, and the base soils by numbers.

$\Delta S_f + \Delta S_b = 0$  was defined on the basis of the assumption that in the limit state two empty size fractions do exist between the filtering fraction and the filtered fraction.

### 3.4. Segregation rule

For the segregation rule, the simple  $e_{max}$  test was used in a modified form. In the  $e_{max}$  tests, the soil is poured into a funnel and flow is allowed from the funnel into a cylinder (10 cm in height and 10 cm in diameter). The segregation test was made with about double the quantity that expectedly filled the 10 cm diameter, 10 cm high cylinder. The funnel is just rising above the soil surface.



**Figure 10.** The filter rule with the safe and unsafe areas. The soils are shown in Figures 8 and 9 [15, 21].



The artificial mixtures of natural sand grains used for the segregation rule were partly continuous mixtures (A, B), partly gap-graded mixtures (C, D, E), as shown in Ref. [21]. The results are shown in **Figure 11**. The difference in the initial and the poured base entropy  $S_0$ , entropy increment  $\Delta S$ , and grading entropy  $S$  was represented in the function of the relative base entropy  $A$ . According to the results, some segregation was always measured, but it was with minor significance if the relative base entropy of the soil was within the following limits:

$$0.4 \leq A \leq 0.7 \quad (21)$$

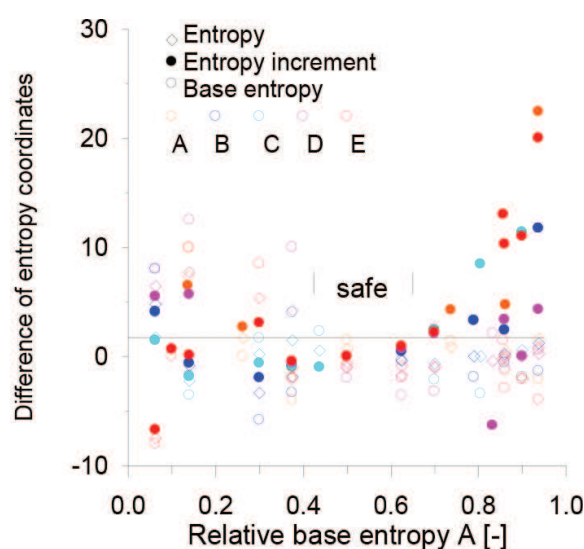
The results showed that the base entropy difference  $S_0$  was negative and the entropy increment difference  $\Delta S$  was positive above the lower limit  $A = 0.4$ . Physically, the proportion of the large grains and the mean  $\log_2$  diameter is larger if the base entropy difference  $S_0$  is larger and vice versa. The value of  $\Delta S$  is larger if the mixture is more uniform in terms of fraction relative frequencies.

### 3.5. Applications

#### 3.5.1. Non-segregating mixtures

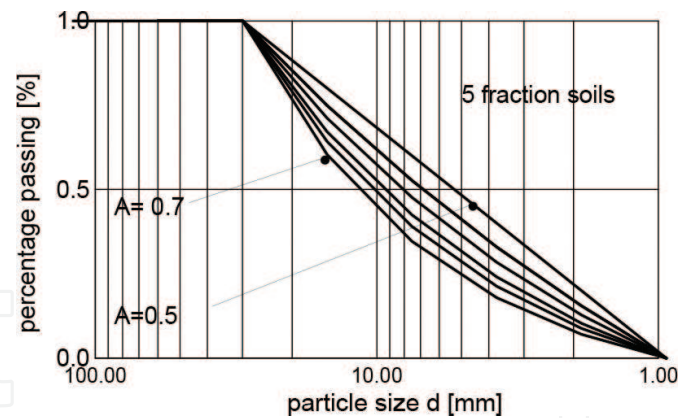
Minimal segregation occurs, and relatively uniformly textured body of soil is achieved for laboratory testing of granular materials or for the earth works, if a non-segregating mixture with  $0.4 < A < 0.7$  is used.

To construct continuous, non-segregating mixtures, some optimal limit curves can be determined. The optimal mixtures computed by a simple algorithm fulfilling Eqs. (15)–(17) for fixed  $N = 5$  (where  $B$  is a maximum for a given  $A$  are for five fractions) are shown in **Figure 12** and **Table 2**. Similar limit curves can be reproduced for any fraction number.



**Figure 11.** Results of the segregation test, mixtures A–E are shown in [21].





**Figure 12.** Limit curves for non-segregating optimal 5-fraction mixtures [21].

$A$ [-]	$a$ [-]	$x_1$ [-]	$\Delta S/\ln N$ [-]
0.50	1.00	0.20	1.44
0.56	1.13	0.15	1.43
0.60	1.23	0.13	1.41
2/3	1.42	0.09	1.34
0.70	1.54	0.07	1.29

**Table 2.** Some non-segregating optimal 5-fraction mixtures.

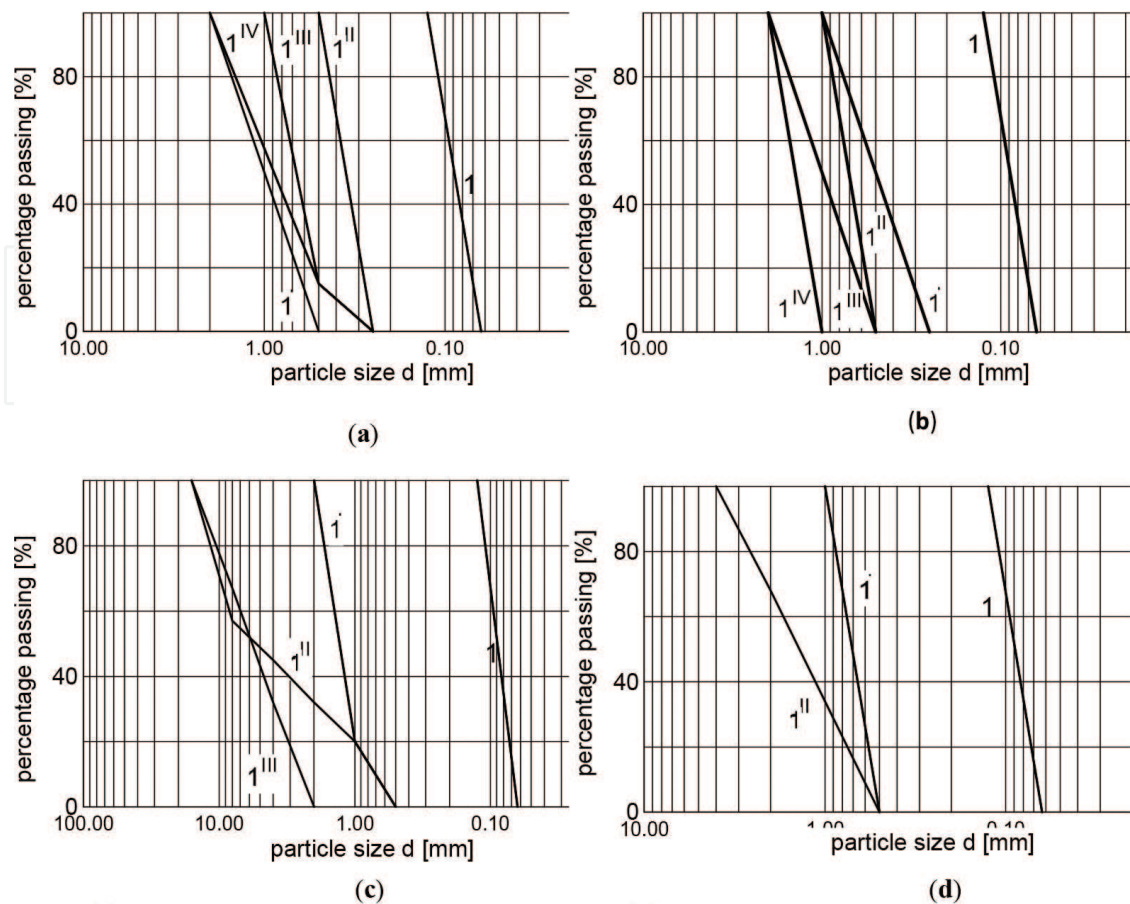
However, it can be noted that soils with gap-graded grading curves can be non-segregating also. For example, in case of a two-fraction soil with gap-graded grading curve, the segregation is minimal if the quantity of the larger fraction varies between 0.4 and 0.7.

### 3.5.2. Testing the filter rules

The grading entropy-based filter law was compared with the existing filtering rules available in the literature. Summaries of well-known filter rules [7, 9–15] for uniformly graded filters and broadly graded filters are presented in Appendix A. These different filtering rules were tested by generating soils with the special-shaped grading curves [21] shown in **Figure 13** and parameterized in **Table 3**.

The 13 combinations listed in **Table 3** were represented for the different filtering rules of the literature, some results are shown in **Figure 14**. If the rule from the literature predicted a successful filtering (i.e., safe behavior), it was plotted with an open circle; where it predicted a failure to filter (i.e., unsafe behavior), it was plotted with a full circle.

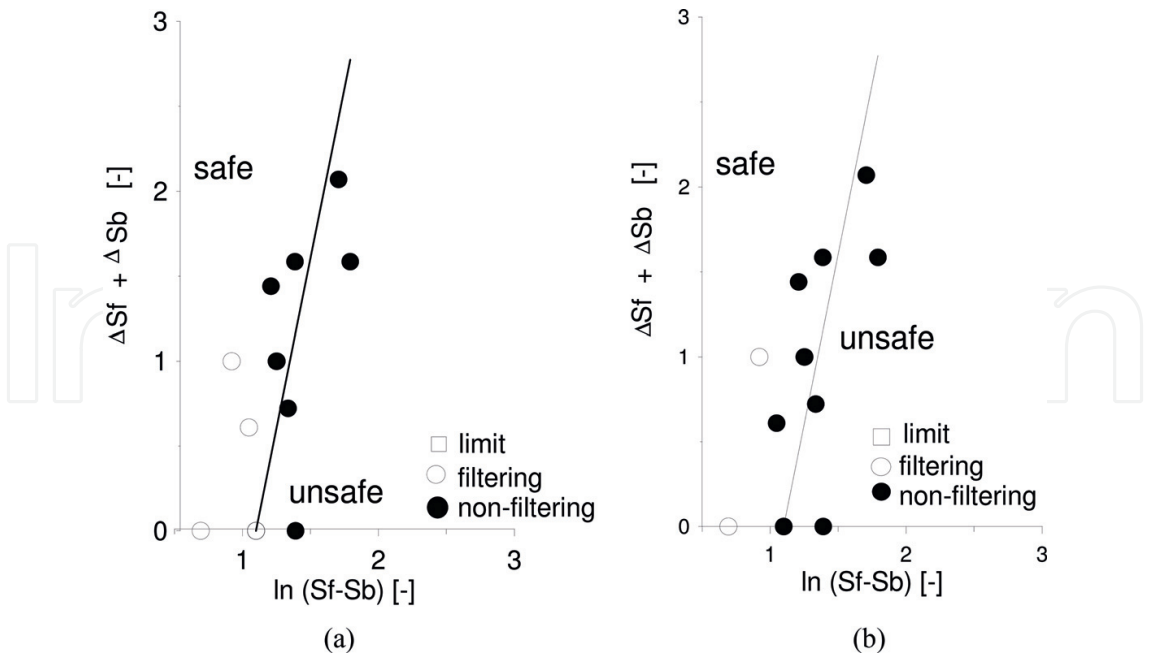
The results indicated that (i) the Terzaghi's filter rule is too conservative, (ii) the Bertram rule is conservative for mixed filters and not acceptable for uniform soils, (iii) the rule of United States Bureau of Reclamation (USBR) for uniform filters is acceptable, and (iv) for mixed filters is not acceptable.



**Figure 13.** The theoretical grading curves used in the testing of existing filtering laws. (a) Mixtures for the Terzaghi's criterion-T. (b) US Bureau simple filters-U. (c) US Bureau mixed filters-UM. (d) Mixtures for Bertram's criterion-B.

		$D_{50}/d_{50}$	$D_{15}/d_{85}$	$D_{10}/d_{60}$	$D_{15}/d_{15}$	$S_{0b}$	$S_{0f}$	$\Delta S_b$	$\Delta S_f$
1	B1-1 <sup>I</sup>	7	4.58	6.5	7.86	13	16	0	0
2	B1-1 <sup>II</sup>	14	5.67	22.5	9.71	13	17	0	1.585
3	T1-1 <sup>I</sup>	10.00	4.17	4.44	7.14	13	16.35	0	1.44
4	T1-1 <sup>II</sup>	4	2.42	3.1	4.14	13	15	0	0
5	T1-1 <sup>III</sup>	6.89	4.17	3.1	7.14	13	15.85	0	0.61
6	T1-1 <sup>IV</sup>	11.11	5.17	6.3	8.86	13	16.5	0	1
7	UM1-1 <sup>I</sup>	13.9	7.3	0.8	12	13	16.8	0	0.722
8	UM1-1 <sup>II</sup>	58	7.3	0.8	12	13	18.51	0	2.07
9	UM1-1 <sup>III</sup>	58	25.5	2.6	40	13	19.	0	1.585
10	U1-1 <sup>I</sup>	5.55	3.02	3.13	4.57	13	15.5	0	1
11	U1-1 <sup>II</sup>	7.78	5.19	5.42	7.86	13	16	0	0
12	U1-1 <sup>III</sup>	11.1	5.85	6.25	8.86	13	16.5	0	1
13	U1-1 <sup>IV</sup>	15.5	10.19	10.63	15.43	13	17	0	0

**Table 3.** The data of the theoretical grading curves used in evaluating the filtration rules, generated for this purpose [15] to four existing filter rules, as shown in **Figure 13**.



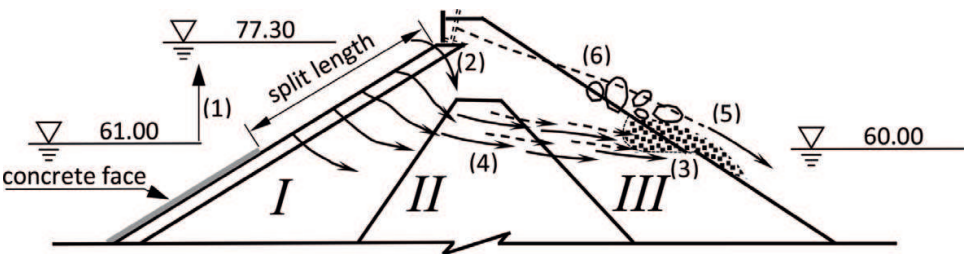
**Figure 14.** Testing some filtering rules, using the theoretical grading curves of **Figure 13** and **Table 3**. (a) The filter rule of USBR for uniform filters [12] is acceptable. (b) The filter rule of Terzaghi [11] is conservative.

#### 4. Case study

Several applications of the entropy-based rules, by examining the reason of piping, softening, dispersive soil behavior, and the goodness of a leachate collection system, were previously presented [19–22]. Here, a dam failure case study is summarized.

The 71 m high, Gouhou rockfill dam was founded on a sandy gravel base layer (**Figure 15**). The dam body consisted of the following parts: the upstream face was a thin layer of material with a design particle diameter of 100 mm, zone I was a transition zone with the maximum diameter of 400 mm, zones II and III were the main rockfill with maximum diameters  $d$  of 600 and 800 mm.

The dam failed [22, 27, 28], killing 288 people, immediately after the first rising water level, and infiltrating the water into the dam body causing internal erosion, piping, and washout of material (see 1–6, **Figure 15**).



**Figure 15.** The Gouhou dam failure. Cross section and failure mechanism.

The relative base entropies of the soils in zones I, II, and III were 0.42, 0.55, 0.58, respectively, all less than  $2/3$  and non-segregating. This result explains why the rockfill material was incapable of forming a stable skeleton of coarse fragments. It follows that the grading entropy-based soil behavior rules would have been capable of predicting piping failure in the Gouhou dam.

## 5. Discussion and conclusion

### 5.1. Some comments on the entropy parameters

The grading entropy parameters are some kind of integrals of the whole grading curve. The same shaped grading curve has the same  $A$ ,  $\Delta S$ , or  $B$  value independent of the value of the minimum grain size. Therefore, these are well-defined parameters and have some physical contents, as follows.

The relative base entropy parameter  $A$  has a potential to be a grain structure stability measure possibly based on the simple physical fact that if enough large grains are present in a mixture, then these will form a skeleton.

The entropy increment  $\Delta S$  measures how much the soil behavior is really influenced by all of its  $N$  fractions. For those grading curves, in which all  $N$  fractions are well represented, the entropy increment is typically close to  $\ln N / \ln 2$ .

The entropy parameters are pseudo-metrics. The difference between base entropies of the filter and base soils,  $S_{0f} - S_{0b}$  describes the distance between the mean diameters. The sum of filter and base soil entropy increments  $\Delta S_f + \Delta S_b$  expresses the sum of the two “effective”  $N$  values (i.e., the total number of important fractions in the two grading curves).

### 5.2. Some comments on the rules

The overall soil stability—according to the experimental results—is described by the criterion that  $A > 2/3$ . In soils which meet this criterion, the matrix of coarser soil particles is stable and able to form a resistant skeleton, even though suffosion may occur.

Some questions arise, for example, in regard to the stability of a single fraction which does not lie in a unique position on the entropy diagram. Since the change due to degradation is the appearance of smalls, which causes an increase in the  $A$  value [29], the one fraction case is likely on the safe side.

Another question is related to the probability that an arbitrary  $N$ -fraction soil is stable. This can be characterized by the relative size of the grading curve space separated with the  $A = 2/3$  hyperplane on condition that the probability is the same in the whole simplex. This number is decreasing with the fraction number (e.g., for  $N = 2$ , the  $1/3$  part of the grading curve space is safe, for  $N = 3$ , the  $2/9$  part of the possible grading curves are safe, for  $N = 18$ , less than the 0.01 part of the possible grading curves are safe).

Significant segregation is unlikely to occur, if the relative base entropy  $A$  is between the limits of 0.4 and 0.7. It is important to note that the same parameter—the relative base

entropy A—is responsible for overall soil stability. Soils which meet both criteria may constitute very small part of the grading curve space and may need careful design in case of broadly graded soils.

The filtration problems are safely solved in the literature for uniform filters and bases (i.e., soils to be protected by the filters). The suggested filter rule can be used to design broadly graded filters (e.g., for clay cores or for leachate collection systems). However, the rule was estimated on the basis of one data point only at the range of very large  $N$  and  $\Delta S$  values.

The testing of the existing rules known from the literature was possible on the basis of the suggested filter rule and using some theoretical grading curves. According to the results, the Terzaghi filter rule is too conservative. The filtering rule of USBR for uniform filters is acceptable. The mixed filter rule of the USBR is not conservative and is not acceptable. The Bertram rule is be conservative for mixed filters and not acceptable for uniform soil.

### 5.3. The importance, use, and implementation of the rules

Applications of the derived entropy-based rules were presented by examining the reason of a dam piping failure, dike piping, dispersive soils, leachate collection system case studies [19–22], a dam example is presented here only. On the basis of the case study, it is apparent that the grading entropy-based soil behavior rules would have been capable of predicting piping failure in the Gouhou dam.

The grading entropy-based criteria can easily be implemented into any laboratory test evaluation software. A basic requirement for the use is that the grading curve information is reliable. The simple soil tests presented here were made on coarse material and the rules apply for soils where the solid fraction is composed of non-clay minerals.

For clay minerals, the same criteria may be valid if the grading curve information is reliable and the appropriate degree of particle agglomeration is reflected in the measurements [30–33]. The first results indicate that the same criteria may be valid for silty soils if the grading curve information is reliable (see e.g., the dispersive soil case studies [20]).

## Appendix A

### Rules for uniform filters

U.S. Bureau of Reclamation [12]:

$$\frac{D_{50}}{d_{50}} = 5 - 10 \quad (22)$$

Sichard [9]:

$$\frac{D_{50}}{d_{50}} = 3 - 4.5 \quad (23)$$

Sherard et al. [7]:

$$\frac{D_{15}}{d_{85}} < 9 \quad (24)$$

where  $D$  and  $d$  denote the filter and the base soil, respectively.

Rules for broadly graded soils

Terzaghi's [10, 11]:

$$\frac{D_{15}}{d_{85}} \leq 4, \frac{D_{15}}{d_{15}} \geq 4 \quad (25)$$

US Bureau of Reclamation [12]:

$$\frac{D_{50}}{d_{50}} = 12 - 58, \frac{D_{15}}{d_{15}} = 12 - 4 \quad (26)$$

Bertram [13]:

$$\frac{D_{15}}{d_{85}} \leq 5, \frac{D_{15}}{d_{15}} = 5 - 9 \quad (27)$$

Cistin [14]:

$$\frac{D_{10}}{d_{60}} < 5, U_D = \frac{D_{60}}{D_{10}} < 5 \quad (28)$$

where  $D$  and  $d$  are diameters of the filter soil and the base soil.

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## References

- [1] Bonelli S, Nicot F, editors. *Erosion in Geomechanics Applied to Dams and Levees*. London Wiley-ISTE; 2013
- [2] Van Beek VM, Knoeff H, Sellmeijer H. Observations on the process of backward erosion piping in small-, medium- and full-scale experiments. *European Journal of Environmental and Civil Engineering*. 2011;**15**:1115-1137
- [3] Sellmeijer H, de la Cruz JL, van Beek VM, Knoeff H. Fine-tuning of the backward erosion piping model through small-scale, medium-scale and IJkdijk experiments. *European Journal of Environmental and Civil Engineering*. 2011;**15**:1139-1154
- [4] Khomenko, V.P. Suffosion hazard: Today's and tomorrow's problem for cities. In *Proceedings of IAEG2006, Nottingham, UK, 6-10 September 2006*; No. 577, p. 8-18
- [5] Lubockov EA. The calculation of suffosion properties of non-cohesive soils when using the non-suffosion analogue. In: *Proceedings of the International Conference on Hydraulic Research*; Brno; 1965. pp. 135-148
- [6] Kenney TC, Lau D. Internal stability of granular filters. *Canadian Geotechnical Journal*. 1985;**22**:215-225
- [7] Sherard JL, Dunningan LP, Talbot JR. Basic properties of sand and gravel filters. *Journal of Geotechnical Engineering*. 1984;**110**:684-700
- [8] Cedergren HR. Seepage control in earth dams. In: *Embankment-Dam Engineering*. New York: Wiley; 1973. pp. 21-45
- [9] Schuler U, Brauns J. Behaviour of coarse and well-graded filters. *Filters in Geotechnical and Hydraulic Engineering: Proceedings of the 1st international conference 'Geo-filter', Karlsruhe, Germany, 20-22 October. 1993* by J. Brauns (Editor), M. Heibaum (Editor), U. Schuler (Editor), CRC Press. Karlsruhe, Germany, pp. 3-18
- [10] Fannin J, Terzaghi K. From theory to practice in geotechnical filter design. *Journal of Geotechnical and Geoenvironmental Engineering*. 2008;**134**:267-276
- [11] Terzaghi K, Peck RB, Mesri G. *Soil Mechanics in Engineering Practice*. New York: Wiley; 1996
- [12] US Bureau of Reclamation. *Earth Manual Part 1*. 3rd ed. Denver: U.S. Department of the Interior Bureau of Reclamation. Geotechnical Research Technical Service Center; 1998
- [13] Bertram GE. *An Experimental Investigation of Protective Filters*. Harvard Soil Mechanics Series No. 7. Cambridge: Graduate School of Engineering, Harvard University; 1940
- [14] Cistin, J. Zum Problem mechanischer Deformationen nichtbindiger Lockergesteine durch die Sickerwasserströmung in Erddämmen. *Wasserwirtschaft Wassertechnik*. 1967;**2**:45-49 (In German)

- [15] Lőrincz J. Grading entropy of soils. Budapest: University Doctoral Thesis, Technical University of Budapest; 1986 (In Hungarian)
- [16] Singh VP. Entropy Theory in Hydraulic Engineering: An Introduction. ASCE Press; 2014. ISBN 978-0-7844-1272-5
- [17] Lőrincz J. On particle migration with the help of grading entropy. Filters in Geotechnical and Hydraulic Engineering: Proceedings of the 1st international conference 'Geo-filter', Karlsruhe, Germany, 20-22 October, 1993 by J. Brauns (Editor), M. Heibaum (Editor), U. Schuler (Editor). CRC Press. Karlsruhe, Germany. pp. 63-66
- [18] Lőrincz J. On granular filters with the help of grading entropy Filters in Geotechnical and Hydraulic Engineering: Proceedings of the 1st international conference 'Geo-filter', Karlsruhe, Germany, 20-22 October. Karlsruhe, Germany, 1993 by J. Brauns (Editor), M. Heibaum (Editor), U. Schuler (Editor). CRC Press. pp. 67-70
- [19] Imre E. Characterization of dispersive and piping soils. In: Proceedings of XI. European Conference on Soil Mechanics and Foundation Engineering, Danish Geotechnical Society Copenhagen. Vol. 2; Copenhagen. 1995. pp. 49-55
- [20] Imre E, Lőrincz J, Szendefy J, Trang PQ, Nagy L, Singh VP, Fityus S. Case studies and benchmark examples for the use of grading entropy in geotechnics. Entropy. 2012;**14**:1079-1102
- [21] Lőrincz J, Imre E, Fityus S, Trang PQ, Tarnai T, Talata I, Singh VP. The grading entropy-based criteria for structural stability of granular materials and filters. Entropy. 2015;**17**: 2781-2811
- [22] Imre E, Nagy L, Lőrincz J, Rahemi N, Schanz T, Singh VP, Fityus S. Some comments on the entropy-based criteria for piping. Entropy. 2015;**17**:2281-2303
- [23] Korn GA, Korn TM. Mathematical Handbook for Scientists and Engineers. 2nd ed. New York: McGraw-Hill; 1975
- [24] Einav I. Breakage mechanics—Part I: Theory. Journal of the Mechanics and Physics of Solids. 2007;**55**(3):1274-1297
- [25] Kézdi Á. Phase Movements in Granular Soils. Notes of Budapest University of Technology and Economics, Graduate Courses. Budapest: Budapest University of Technology and Economics; 1975 (manuscript)
- [26] O'Sullivan C. Applying micro-mechanical analysis to realistic sands. In: Proceedings of the International Symposium on Discrete Element Modelling of Particulate Media: In Celebration of the 70 Birthday of Colin Thornton, Birmingham, 29-30 March 2012. Royal Society of Chemistry Publishing. London.
- [27] Zhang LM, Chen Q. Seepage failure mechanism of the Gouhou rock-fill dam during reservoir water infiltration. Soils Found. 2006;**46**:557-568
- [28] Gouhou Dam Failure Investigation Team. Technical details of the Gouhou Dam. In: Gouhou Concrete-Faced Rockfill Dam—Design, Construction, Operation, and Failure. Beijing: Water Conservancy and Hydropower Press; 1996. pp. 111-245

- [29] Lőrincz J, Imre E, Trang PQ, Telekes G, Juhász M, Fityus S. Grading entropy and degradation of sands. In: XV Danube - European Conference on Geotechnical Engineering. Vienna. 9-11. September 2014. *Austrian Society of Engineers and Architects, ÖIAV*. pp. 717-722
- [30] Emerson WW. A classification of soil aggregates based on their coherence in water. *Australian Journal of Soil Research*. 1967;5:47-57
- [31] Rengasamy P, Greene RSB, Ford GW. The role of clay fraction in the particle arrangement and stability of soil aggregates—A review. *Clay Research*. 1984;3:53-67
- [32] Yong RN, Amar IS, Harald PL, Jorgensen MA. Interparticle action and rheology of dispersive clays. *Journal of Geotechnical Engineering*. 1979;105:1193-1211
- [33] ASTM 2005 C136-06. Standard Test Method for Sieve Analysis of Fine and Coarse Aggregates. West Conshohocken: American Society for Testing and Materials; 2003